Assignment: Section 5.3: 2 (a,b), 8, 24, 26a, 28a, 32a  ( 7th edition)

Assignment: Section 5.3: 12, 26c ("5 | a+b" meas that 5 divides a+b), 43, 44; ( 7th edition)

**2.** Find *f (*1*)*, *f (*2*)*, *f (*3*)*, *f (*4*)*, and *f (*5*)* if *f (n)* is defined recursively by *f (*0*)* = 3 and for *n* = 0*,* 1*,* 2*, . . .*

**a)** *f (n* + 1*)* = −2*f (n)*.

**-6, 12, -24, 48, -96**

**b)** *f (n* + 1*)* = 3*f (n)* + 7.

**16, 55, 172, 523, 1576**

**8.** Give a recursive definition of the sequence {*an*}, *n* =

1*,* 2*,* 3*, . . .* if

**a)** *an* = 4*n* − 2.

**Basis Step: A(1) = 4 - 2 = 2**

**Recursive Step: A(n+1) = 4(n+1) - 2**

**= 4n + 4 - 2**

**= 4n - 2 + 4**

**= an + 4**

**b)** *an* = 1 + *(*−1*)n*.

**Basis Step: A(1) = 1+(-1)1 = 0**

**Recursive Step: A(n+1) = 1 + (-1)n+1**

**= 1 + -1n ( -11)**

**= 1 + (((-1)n + 1) -1) -1))**

**=1 + (an - 1)(-1)**

**=2 - an**

**c)** *an* = *n(n* + 1*)*.

**Basis Step: A(1) = 1(1+1) = 2**

**Recursive Step: A(n + 1) = n+1(n+1 + 1)**

**= n (n+1) + n + n+1 + 1**

**= an 2n + 2**

**d)** *an* = *n*2.

**Basis Step: A(1) = (1)2 = 1**

**Recursive Step: A(n+1) = (n + 1)2**

**= n2+2n+1**

**=an + 2n + 1**

**24.** Give a recursive definition of

**a)** the set of odd positive integers.

Basis Step: 1∈ S

Recursive Step: If x ∈ S, then x + 2 is ∈ S

**b)** the set of positive integer powers of 3.

Basis Step: 3 ∈ S

Recursive Step: If x is ∈ S, then x3 ∈ S

**c)** the set of polynomials with integer coefficients.

Basis Step: 0 ∈ S

Recursive Step: If p(x) ∈ S, then p(x) + cxn ∈ S, where c ∈ Z, n ∈ Z, and n >= 0

**26.** Let *S* be the subset of the set of ordered pairs of integers defined recursively by

*Basis step: (*0*,* 0*)* ∈ *S*.

*Recursive step:* If *(a, b)* ∈ *S*, then *(a* + 2*, b* + 3*)* ∈ *S* and *(a* + 3*, b* + 2*)* ∈ *S*.

**a)** List the elements of *S* produced by the first five applications

of the recursive definition.

(2, 3) (3, 2)

(4, 6) (6, 4) (5, 5)

(6, 7) (9, 9) (7, 8) (8, 7) (9, 6)

**28.** Give a recursive definition of each of these sets of ordered

pairs of positive integers. [*Hint:* Plot the points in the set

in the plane and look for lines containing points in the

set.]

**a)** *S* = {*(a, b)* | *a* ∈ **Z**+*, b* ∈ **Z**+*,* and *a* + *b* is odd}

**32. a)** Give a recursive definition of the function *ones(s)*,

which counts the number of ones in a bit string *s*.

In Exercises 12–19 *fn* is the *n*th Fibonacci number.

**12.** Prove that *f* 2 1+ *f* 22 +· · ·+*f* 2 *n* = *fnfn*+1 when *n* is a positive integer.

**26.** Let *S* be the subset of the set of ordered pairs of integers

defined recursively by

*Basis step: (*0*,* 0*)* ∈ *S*.

*Recursive step:* If *(a, b)* ∈ *S*, then *(a* + 2*, b* + 3*)* ∈ *S* and *(a* + 3*, b* + 2*)* ∈ *S*.

**c)** Use structural induction to show that 5 | *a* + *b* when *(a, b)* ∈ *S*.

**43.** Use structural induction to show that *n(T )* ≥ 2*h(T )* + 1, where *T* is a full binary tree, *n(T )* equals the number of vertices of *T* , and *h(T )* is the height of *T* . The set of leaves and the set of internal vertices of a full binary tree can be defined recursively.

*Basis step:* The root *r* is a leaf of the full binary tree with exactly one vertex *r*. This tree has no internal vertices.

*Recursive step:* The set of leaves of the tree *T* = *T*1 · *T*2 is the union of the sets of leaves of *T*1 and of *T*2. The internal vertices of *T* are the root *r* of *T* and the union of the set of internal vertices of *T*1 and the set of internal vertices of *T*2.

**44.** Use structural induction to show that *l(T )*, the number of leaves of a full binary tree *T* , is 1 more than *i(T )*, the number of internal vertices of *T* .